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# Individual Differences in Distribution Strategy of the Working Memory Resources during Space Geometry

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*Fifteen high-achieving and 15 low-achieving students were selected from 400 second graders. The distribution strategy of the working memory resources in the process of solving the geometry problems was explored. Results showed that: (1) as problem difficulty increased, the first internal thinking time increased as well. There were significant differences in solving difficult problems and middle-level problems between high-achieving students and low-achieving students, but them; (2) as problem difficulty increased, frequency of external-strategy used by high-achieving students increased, and that used by low-achieving students decreased. This suggests that compared to low-achieving students, high-achieving students were able to distribute working memory resources in a more rational way.*

***Key words:*** cognitive load, working memory, individual difference, distribution strategy.

## Introduction

Working memory is a concept put forward by Baddeley (2002), which is used to describe the temporary storing and processing of information. It is a continuous working state formed as the information changes in the processing of cognition. Apart from the short-term working memory for temporary information storage, there is also long-term working memory based upon long-term memory and can be skillfully utilized by the operator. Information in the long-term working memory can be stably stored for a long time, and can also establish a short-term retrieval passage through retrieval cues in the short-term working memory. The long-term working memory is virtually capable of retrieving and storing the information in the long-term human working memory in a rapid and reliable manner; it is reflected in the familiar cognitive activities that people are engaged in. This capability can be obtained through training or long term practice. However, only when supported by the short-term working memory, can the long-term working memory be brought into fully play. Overall speaking, the working memory is a requisite sort of temporary storing and processing of information when one executes advanced cognitive tasks, and it plays an extremely important role in many complicated cognitive activities, including reasoning, language understanding, learning, and mental arithmetic. In the meantime, the two features of working memory, i.e. limited capacity and relatively short term of information storage, have greatly restrained people's complicated cognitive activities (Baddeley, 2002; Baddeley, 1992). Therefore, when the cognitive task load comes to a certain point, it will become difficult to keep the information and further process the tasks by solely relying upon the internal strategy of one's brain. It requires an individual to transfer part of the cognitive activities to the exterior by means of some strategy in order to overcome the limited internal working memory and maintain the information required in the processing of cognition (Yang, Go, Wang, & Chen, 2004). As a result, the studies of distribution strategy of the working memory resources are generated. Strategy refers to a method, and there are three major types of distribution strategies of the working memory resources: external strategy, internal strategy, and internal-external strategy. Studies have shown that external strategies (e.g. marking, and writing down math equations and marks, etc.) use external space to store the information; internal strategies (e.g. language rehearsal) utilize internal space to keep the information; internal-external strategies draw support from both internal and external space to maintain the information.

Carlson, et al., carried out an early study on the distribution strategy of working memory resources (Carlson, Wenger, & Sullivan, 1993; Cary & Carlson, 1999; Cary & Carlson, 2001). For instance, Cary and Carlson (2001) investigated the choice of internal and external resource distribution of college student working memory for the first time; Liu and Wo (2003) studied 274 middle school students in terms of the distribution strategy of middle school student working memory resources, with the cognitive operation of serial numbers as the experimental materials; Cao, Liu and Lin (2005) studied the effect of cognitive load upon the distribution strategy of primary school student working memory. These studies on elementary algebraic operations have demonstrated that the distribution strategy of working memory resources can be divided into a number of specific operation modes; the application of strategies is varied; there is a sharp contrast in the reaction time of primary school students when they use different strategies to solve problems; students from higher grades are more inclined to opt for more effective strategies than those younger; and the difficulty of cognitive load has apparent impact on the distribution strategy of primary school student working memory resources. Up to now, most studies concerning the distribution strategy of working memory resources are concentrated on the elementary algebraic cognition, while few researchers have explored the field of geometry.

Based on a large amount of faithful records, investigation, and analysis of the external behaviors of high school second graders solving space geometry problems, this paper proposes four categories and definitions of the distribution strategy of working memory resources when the high school students solve space geometry problems; it attempts to study their selection modes and application features in a dynamic perspective through a thorough, real record of the problem-solving process, such as the time of application and the frequency of conversion between various strategies, the difference in strategy application of the same individual under different cognitive loads, and that of different individuals under the same cognitive load. It aims to study the disparities of the distribution strategy of working memory resources between high-achieving students and low-achieving students in different cognitive activities, trying to shed some light on the teaching and learning of high school geometry.

## Methods

The experiment adopts a two (student types: high-and low-achieving students in math) and three (three levels of difficulty: easy problems, middle-level problems, and difficult problems) repeated measurement design. The student type is the between-participants factor, and the problem difficulty is the within-participants factor.

### Participants

Fifteen high-achieving students and 15 low-achieving students were selected based on their math grade from a pool of 400 second graders at a high school in a large metropolis in north of China.

## Experiment Materials

The strategy study of solving space geometry problems requires the participants to complete a written exam, of which the materials are from the 2003 and 2004 college entrance examination papers of Beijing. Each problem includes three sub-problems, and the materials fall into three levels of difficulty (easy, middle-level, difficult) in accordance with the order of the sub-problems.

## Procedure

After we have a secured a laboratory, set up a video camera, and asked the participants to take the exam one by one; we determined the types of distribution strategies used by these participants by analyzing the students' problem-solving performance recorded by the camera; gathered the statistics concerning the frequency and the amount of time the participants used in solving each problem, and filled in the statistics table.

The paper divides the strategies used by the participants in solving space geometry problems into four categories, namely, internal thinking strategy, external marking strategy, external diagrammatic strategy, as well as external writing and calculating strategy. The internal thinking strategy is characterized by thinking about, calculating and solving the problems in one's mind without writing on the paper; external marking strategy is reflected in one's marking symbols, letters, relations, length, etc. onto a diagram; one using the external diagrammatic strategy usually draw auxiliary lines in the diagram, or draw anther diagram on the answering sheet; the external writing and calculating strategy, however, is characterized by calculating the problem on the paper, or writing down the calculating and reasoning process. In compliance with the categorization of behavior in solving geometry problems, this categorization of strategy has no confusing factors, and is easy to be distinguished.

## Data Analysis

The internal thinking strategy is timed, with second as the unit. When the conversion between internal and external strategies is timed, if one stops writing for more than five seconds, he/she will then be considered as executing the internal thinking strategy, which will also be noted as a strategy conversion; the total time of internal thinking strategy refers to a combination of time used when solving a sub-problem through the internal thinking strategy. The external marking strategy and diagrammatic strategy are measured by strokes, with each stroke being noted as one time of adopting the strategy; the external writing and calculating strategy is measured by frequency, among which the calculating strategy is measured by the number of equations, with one "equal sign" as one time of strategy using; the writing strategy of reasoning is measured by the number of causal relations, with one pair of causal relations ("∵ ...∴ ...") counted as two times using a strategy. The total time of problem solving refers to the time when the participants begin to solve the problem until he is finished. Those problems which cannot be solved are subject to the time when the participant declares to give up.

## Results

### First-internal-thinking Time under Different Cognitive Loads

The descriptive statistics in Table 1 showed the different first-internal-thinking time between high-achieving students and low-achieving students in solving mathematics problems. The first-internal-thinking time of the former (26seconds) was significantly shorter than that of the latter (52seconds).

*Table 1*

**Descriptive Statistics Results (SD) of First-internal-thinking Time(s)**

| | High-achieving students | | Low-achieving students | |
|---|---|---|---|---|
| | *Mean* | *SD* | *Mean* | *SD* |
| Easy | 23.7s | 8.7s | 33.5s | 22.9s |
| Middle-level | 20.2s | 6.5s | 43.0s | 17.7s |
| Different | 33.9s | 16.5s | 80.5s | 35.9s |

The within-participants factors suggested (see Table 2): the difficulty of mathematical problems boasted considerable main effect ($F(2,56)=17.51, p=0.000$), significant interaction effect ($F(2,56)=6.17$, $p<0.01$) existed between the difficult questions and the student type. The between-participants factors suggested: the student type had obvious main effect.

The interaction effect of Difficulty of Mathematical Problems × Student Types indicates the distinct influences such difficulty had on the first-internal-thinking time of high and low achieving students. The inspection afterwards discovered: with easy problems, there was no significant difference ($F(1,29)=2.38$, $p>0.05$) in the first-internal-thinking time between high-achieving students and low-achieving students, however, marked differences $F(1,29)=20.86$, $p<0.001$) arose in that of solving difficult problems between the two types of students. The above aspects demonstrated that no significant first-internal-thinking time difference existed between high and low achieving students in case of easy problems, as to middle-level and difficult problems, however, the time that high-achieving students spent was observably shorter than that of low-achieving students

*Table 2*

**The Result of Repeated Measure ANOVA**

| Sources | *SS* | *Df* | *MS* | *F* |
|---|---|---|---|---|
| Problem Difficulty | 14841.27 | 2 | 7420.64 | 17.51**** |
| Problem Difficulty × Student Type | 5230.72 | 2 | 2615.36 | 6.17** |
| Error | 23723.18 | 56 | 423.62 | |

Note: **< 0.01; *** < 0.001

**Frequency of External Strategy**

The external strategy frequency is short for the average frequency of external identification, external graphic and external writing and calculation strategy frequencies calculated in the process of solving two easy, middle-level and difficult problems by the participants. The descriptive statistics in Table 3 suggested that in general a lower frequency of external strategy was adopted to solve easy problems whereas such frequency significantly increased in middle-level and difficult problems. However, low-achieving students displayed greater frequency in the application of external strategy when solving easy and middle-level problems, this practice is decreased when solving difficult problems.

*Table 3*

**Descriptive Statistics Results of External Strategy Frequency**

| | High-achieving students | | Low-achieving students | |
|---|---|---|---|---|
| | *Mean* | *SD* | *Mean* | *SD* |
| Easy | 19.2s | 2.2 | 37.6s | 2.2 |
| Middle-level | 31.7s | 1.8 | 38.9S | 1.8 |
| Difficult | 30.4s | 2.1 | 19.1S | 2.1 |

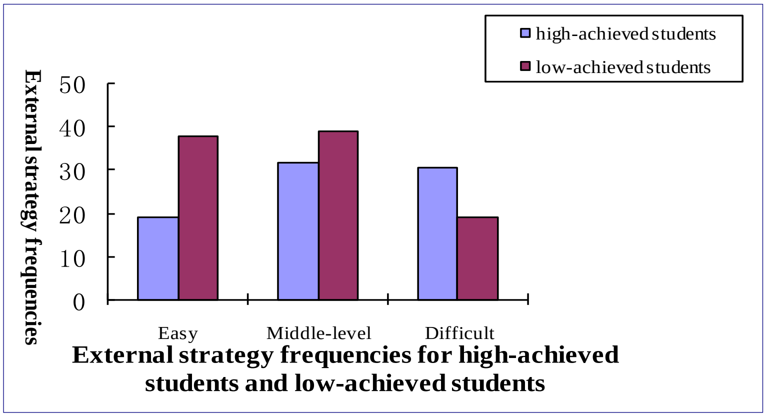


*Figure 1*. **External strategy frequencies for high-achieving students and low-achieving students.**

The Wilcoxon Signed Ranks Test conducted on the frequencies of two types of participants applying external strategy indicated that: significant differences (*Z*=—4.174, *p*<0.001; *Z*=2.494, *p*<0.05; *Z*=3.118, *p*<0.01) existed between the two frequencies of external strategy adopted by high-achieving and low-achieving students in solving easy, middle-level and difficult problems, respectively. These results demonstrated that compared to their high-achieving peers, low-achieving students, applied the external strategy more frequently to solve easy and middle-level problems. On contrast, the high-achieving students used the external strategy more frequently to solve difficult problems.

Implemented on the external strategy frequency for each subject under three categories of problems with respective difficulties, the Wilcoxon Signed Ranks Test revealed: those high-achieving students used more external strategies (*Z*=3.125， *p*<0.01) in solving middle-level problems rather than easy ones, and the external strategy frequency experienced no significant variation ($Z$=1.037， $p$>0.05) in dealing with middle-level and difficult problems, which is the same as to the low-achieving students in dealing with middle-level and easy problems, however, less external strategies were employed in solving difficult problems than those employed in middle-level ones.

## Discussion and Conclusion

The research results in Table 1 showed that as problem difficulty increased, the first-internal-thinking time for the low-achieving students rose from 33.5s, 43.0s to 80.5s in the process of space geometry problem solving. This suggests that when the mathematical problems became more difficult, the low-achieving students would simply first add more internal-thinking time instead of actively resort to the problem solving mode of external strategy. Compared to easy and middle-level problems, the speed for initiating external strategy underwent a considerable decrease upon difficult problems. On the other hand, the first-internal-thinking time for the high-achieving students varied slightly form 23.7s, 20.2s up to 33.9s, respectively, and all the statistics concerned were lower than that of the low-achieving ones, which manifested the relatively little influence exerted by problem difficulty on the high-achieving students' internal-thinking time. In addition, they were supposed to possess more substantial internal resources and larger working memory capacity and smoother access to information, meanwhile, the prompt application of external strategy could effectively decompose the pressure of working memory capacity resulted from the raised cognitive load so as to solve and analyze problem in a swifter way with more accurate methods.

Although the available time of the first internal strategy for high-achieving students increased and the startup speed slightly decreased, still, they had remarkable advantages over the low-achieving ones when solving difficult problems. The early mobilization and application of external strategy fully demonstrated that, in the process of solving comprehensive and difficult problems, high-achieving students not only obtain internal resources support, but also take advantage of external strategy to help them work out the problems in an active and rational way. Meanwhile, the low-achieving students could hardly find the key to the problem due to their deficiency of internal resources and inefficiency in acquiring the necessary information. Moreover, their failure in decreasing the problem difficulty through prompt and effective adoption of external strategy made it difficult to find a rapid and reasonable solution.

The research results in Table 3 suggested that the frequencies of high-achieving students using the external strategy were 19.2, 31.7 and 30.4, respectively, when solving easy, middle-level and difficult problems. Meanwhile, the frequencies for the low-achieving students amounted to 37.6, 38.9 and 19.1, respectively, demonstrating a significant difference between the two groups of participants. Specifically, when solving easy and middle-level problems, high-achieving students showed less utilization of external strategy which is an indication that they possessed relatively substantial internal working memory resources in order to solve problems with the assistance of external strategy. In contrast, the deficient working memory resources required low-achieving students to resort to more external strategies. The low-achieve students' over-use of those external strategies upon easy and middle-level problems not only demonstrated their speed and skill shortage in extracting and integrating internal resources, but also implied the great consumption of internal and external resources, in other words, they should spend more time and energy to correctly solve problems. On that basis, low-achieving students possessed insufficient internal resources to provide solutions and any disturbance in thinking could definitely result in the blind and inefficient usage of external strategies as well as the decrease of its frequency.

The experiment results showed the following: (1) as mathematical problem difficulty increased, the first-internal-thinking time of both two groups of participants tended to increase. However, there were no significant differences in the first-internal-thinking time of solving simple problems between high-achieving students and low-achieving students; high-achieving students tended to use first-internal-thinking strategy when solving middle-level and difficult problems in a comparatively short period of time, while low-achieving students were apt to adopt the aforementioned strategy in a relatively long period of time; (2) As mathematical problem difficulty increased, frequency of external-strategy used by each said group in the process of solving problems presented different characteristics, i.e. frequency of the external-strategy used by high-achieving students was prone to increase as mathematical problem difficulty increased, and they could appropriately increase the utilization of external strategy in a bid to distribute working memory resources. The frequency adopted by the low-achieving students decreased, which was demonstrated in the overuse of external strategy on simple and middle-level problems, all of which illustrated that the high-achieving students could utilize working memory resource in a comparatively reasonable way in comparison to those low-achieving students. As mathematical problem difficulty increased, high-achieving students could shorten the first-internal-thinking time while increasing the utilization of the external strategy, which was conducive to the reduction of working memory load as well as the resolution of the problems. On the contrary, low-achieving students may increase the first-internal-thinking time and reduce the utilization of the external strategy, which would increase the working memory load and hamper the resolution of mathematic problems.

## Educational Implication

### Intensify the Fundamental Teaching

As the long-term working memory can be obtained through training and long-term practice, it is suggested that in teaching geometry in secondary schools, we shall strengthen the study and research of the elementary knowledge and method and typical figure, and unceasingly summarize and combine the knowledge learnt so as to make the said aspects become a part of the long-term memory of the said students; regularly review and reproduce the said aspects so as to make them become the sound extracting and retrieving channel, and thus facilitate the speedy extraction and association of the said students in the process of solving problems.

### Emphasize the Guidance of Strategy

In allusion to such characteristics as the limited capacity of working memory and comparatively short period of time for maintaining information, effective guidance on the utilization of working memory resource strategy of the said students shall be conducted. For example, when reviewing the problems, the students shall adopt external marking strategy and utilize intuitionist marking so as to prevent the known condition memory from occupying the usage space of internal resources. During the process of analyzing problems, the students shall also implement the external writing and calculating strategy and write down concretely the preliminary conclusions obtained from internal thinking, which could further facilitate the promotion and development of thinking. In conclusion, it was expected that the students could appropriately implement the internal and external working memory resources and effectively reduce the internal thinking strength and pressure of the strained and insufficient internal resources so as to improve the techniques of reasonably distributing and utilizing the resources in the process of resolving problems.

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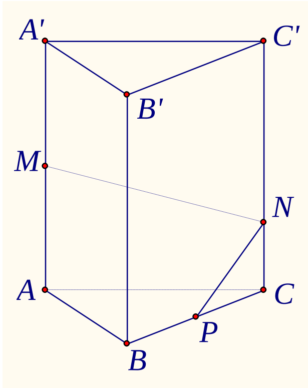
## Appendix

**Question 1:**

As shown in the following figure, in triangular prism *ABC-A'B'C'*, provided *AB*=3,*AA*'=4, *M* refers to the middle point of line *AA'*, *P* is one point on line *BC*, and the shortest route from *P* to *M* along the side of the said prism via *CC*' is $\sqrt{29}$, assume that the intersection point between the said shortest route and *CC*' is *N*, the following aspects shall be deduced:



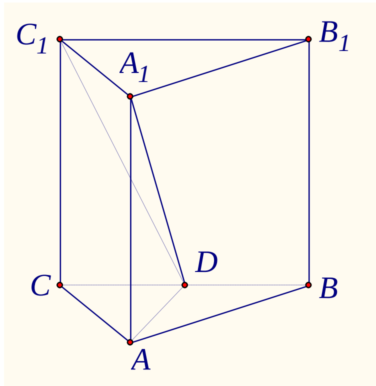

(1) diagonal length of side expansion graph of the said prism;

(2) lengths of *PC* and *NC*

(3) dimension of dihedral angle (acute angle) between planes *NMP* and *ABC*. (expressed by inverse circular trigonometric function)

**Question 2:**

As shown in the following figure, in triangular prism$ABC$-$A_1B_1C_1$, provided *D* is the middle point of line *NC* and *AB*=*a*,




(1) $A_1D \perp B_1C_1$ shall be verified;

(2)distance from point *D* to plane $ACC_1$ shall be deduced;

(3)judge the position relationship between $A_1B$ and plane $ADC_1$ and demonstrate the conclusion.

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