

Concerning the New Mathematics Curriculum: The Pedagogical Content Knowledge of High School Mathematics Teachers

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This paper describes a 2-year longitudinal study into the pedagogical content knowledge of 176 high school mathematics teachers from 83 schools in Jiangsu province. The data was collected through classroom observations, questionnaire and interviews. The results indicated that: (1) the teachers were not well prepared for the new curriculum because they lacked sufficient understanding of the principles, standards, and objectives of the new curriculum; (2) the teachers needed to expand their repertoire of teaching strategies and their knowledge of the newly added contents in the syllabus; and (3) there was a gap between the teachers' pedagogical content knowledge and their teaching practice.

Key words: new curriculum, pedagogy content knowledge, questionnaire, interview, class observation.

Introduction

“To give students a cup of water, a teacher should first have a bucket of water.” This metaphor has clearly conveyed the importance of teachers' knowledge in teaching. Currently, in a turbulent period of mathematics curriculum reform, do our teachers still have the courage to say that they have the “bucket of water”? Even though the bucket of water is there, is it nutritious enough to fulfil the multiple requirements of the new curriculum? Or is there any sustenance missing researchers have to face these fundamental problems in investigating the effects of current mathematics curriculum reform.

Definitely, the mathematics teacher's bucket of water involves a great diversity of knowledge, among which, considering the weight of its influence on mathematics curriculum reform, teachers' content knowledge, knowledge of the curriculum standard, and pedagogical content knowledge are of high importance. Based on this consideration, the study inspected a group of high school teachers' pedagogical content knowledge (PCK) after their

participation in the new mathematics curriculum reform so as to find out their real understanding of the new curriculum and investigate the influence of the new curriculum on their practice.

Methodology

The latest *Mathematics Curriculum Standards for Senior Middle School (Reformed version)* emphasized the conversion of the teaching methodology as well as the contents of the curriculum. This conversion has fluctuated teachers' traditional concept of teaching. Relevant government organizations have made a set of preparations; however, no sufficient material indicates the information about the real situation of the teachers' PCK and whether their knowledge is sufficient for the reform. Therefore, the two purposes of this study are: (1) investigating the state of the teachers' PCK and (2) inspecting the difference between their current state and the state expected by the new curriculum.

Generally, there were great differences between the new mathematics curriculum and the old version. In addition to the division of obligatory courses (OC) and elective courses (EC), the OC has also been divided into five sub-modules and EC into four sections, with the first two sections consisted of several modules and the other two of several topics. Since the third and fourth sections are still under investigation, they have little impact on the new curriculum and our study focuses on the teachers' understanding of the obligatory courses and the first two sections of the elective courses, as well as their understanding of the new concept and teaching method proposed by the new curriculum. Hence, our entire investigation on the design of the new curriculum aimed at the following areas: The introduction to algorithm in OC module 3 and cases of statistic, probability, expansion of number system, and the introduction to plural in section 1 and 2 of EC. Through the investigation of these topics, we could look into teachers' understanding of the new curriculum and their state of professional teaching knowledge. The second purpose of this study could be achieved through an inquiry on teachers' understanding of the new *Mathematics Curriculum Standards for Senior Middle School (Reformed version)* and their practice of the standards.

Referring to the Mathematics textbooks published by Jiangsu Education Press, the entire study was carried out within Jiangsu province. In order to collect more comprehensive data, this study adopted questionnaire, classroom observation, and interviews as the channels for data collection.

Questionnaire

Questionnaire is the most important way of data collection among the three. The samples include Grade 10 and Grade 11 mathematic teachers who used the new textbooks in 10 cities in Jiangsu province, including Nanjing, Suzhou, Yangzhou, Xuzhou, and Yancheng and so on. During the data collection, we paid attention to the difference in different distinct, schools of different levels, teachers' teaching experiences and the title of their posts so as to guarantee the representativeness and effectiveness of the sample.

The questionnaire includes 13 items (see Appendix) and mainly focused on the teachers' PCK under the background of the new curriculum. Among the 13 items, item 1~10 are multiple-choice questions and 11~13 are brief-answer questions. Item 1~6 focused on the update of teachers' concept of teaching while items 7~13 on the update of teachers' knowledge structure. Since teachers' PCK involves many factors, we realized that these 13 questions were not enough for the information we devised though we had a large scale sample.

Classroom Observation

If the questionnaire relates the teachers' self-checking, classroom observation is a kind of assessment on teachers' PCK from an outsider's view. Classroom observation is a channel to observe and analyze teachers' teaching behavior which gives information about the teachers' concepts of teaching and their understanding of the contents in textbooks.

Teaching is a self-organized process for teachers, thus their description afterward could completely reflect what was happening at a certain time. Furthermore, according to the nature of human beings, what a person can tell is always less than what he knows, which means, people may not realize their behaviours stirred up from their subconscious, or they have difficulties describing their behaviours accurately (Polanyi, 1969). Therefore, classroom observation is a supplement to the questionnaire. It will provide a dynamic record of teachers' teaching and obtain the information which could not be easily gained from other channels.

The researcher prepared relevant questions before each classroom observation. These questions were designed according to the characteristics and the focal points of the lesson and they were asked according to what was observed. Within the two or more years (Sep. 2005 to Nov. 2007), starting

from the very beginning of the formal execution of the new textbooks, we observed 46 classes, involving 28 teachers in Grade 10 and Grade 11. These classroom observations covered teachers of different types.

Interview

Interview is a way to validate the thoughts which emerged from questionnaire surveys and classroom observations. The interviews in this study can be categorized into two levels: (1) based on classroom observations of the 28 teachers, the purpose is to understand the reasons or purposes of some of their designs or behaviours in class, together with some general questions about PCK; and (2) with the opportunity of teaching master teachers, the researcher selected 27 teachers from 88 master teachers who use the new textbooks. A detailed interview was executed with each of them.

Besides this, in order to better understand teachers' cognition toward "the state of teachers' PCK", the researcher organized a discussion with 27 teachers and then summarized their sharing characteristics and common problems.

Data Analysis and Results

Results of Questionnaire

At all, 176 out of 220 questionnaires were collected. These 176 questionnaires were from 83 different schools located in Jiangsu province. The distribution of these questionnaires is shown in Table 1 according to the levels of the schools, titles of the teachers, and the range of the teachers' teaching experience.

Table 1
Distribution of the Sample for Questionnaire

	Level of School				Title of Teacher			Teaching Age			
	1	2	3	4	Senior	Secondary	Junior	0~5	6~15	16~25	26~
Distribution of the Sample	46	32	92	6	56	97	23	53	62	42	19

Note. Levels of school: 1= normal school, 2 = two star school, 3 = three star school, 4 = four star school.

The questionnaire has two sections. The first section has 10 items with A, B, C, D, and E as five different choices. The teachers are required to choose only one according to their real situation. Space is provided after each item for teachers to fill in their own opinions if no suitable choice is provided. Among those 176 teachers, 12 teachers gave their opinions for Item 5; 7 teachers did not answer Item 8; 11 teachers provided their opinions for Item 8; 3 teachers chose both B and E for Item 10 and 17 and 5 teachers selected both C and E for Item 10. The other answers were normal as reported in Table 2.

Table 2
Description Data of Questionnaire

No.	A		B		C		D		E	
	Sample	%	Sample	%	Sample	%	Sample	%	Sample	%
1	2	1.1	13	7.4	41	23.3	101	57.4	19	10.8
2	6	3.4	21	11.9	82	46.6	55	31.3	12	6.8
3	5	2.8	37	21.0	67	38.1	13	7.4	54	30.7
4	12	6.8	61	34.7	52	29.5	45	25.6	6	3.4
5	25	15.2	30	18.3	36	22.0	37	22.6	36	22.0
6	20	11.4	32	18.2	45	25.6	42	23.9	35	19.9
7	3	1.7	29	16.5	123	69.9	21	11.9	0	0
8	132	83.5	3	1.9	1	0.6	10	6.3	11	7.0
9	2	1.1	30	17.0	94	53.4	41	23.3	9	5.1
10	18	10.4	97	55.1	31	17.6	15	8.5	12	6.8

Note. Item 5 has 164 samples, Item 8 has 158 samples, and Item 10 has 173 samples; the other items all have 176 samples. Due to rounding off, the sum of percentage may not equal to 100%.

The second part of the questionnaire has 3 brief-answer questions. The first two intend to investigate the depth and width of teachers' mathematical content knowledge while the last one aims to understand teachers' concept of teaching in mathematics. Among the 176 questionnaires, 43 teachers did not provide answers to Item 11, 29 teachers did not answer item 12, and 23 of them did not answer both item 11 and 12. Some "objective" reasons might be "lazy" and "don't want to waste time", but the real reason behind this might be

that they lack relevant knowledge. This possibility could be supported by the answers collected. For example, in item 11, among those 133 questionnaires who provided answers, only 8 teachers could point out that is the famous Bertrand problem, which is a typical question for geometry in the textbook. The situation for item 13 is better for the others, on one hand, all the 17 teachers have answered this item; on the other hand, the teachers have a better understanding about the traditional key concepts and basic methods, like function, vector, operation, the concept of space, and the combination of number and shape, but weaker in the new added contents such as algorithm, statistic, random variability and so forth. This result indicates that the teachers still need more time to appreciate the ideas of the new contents.

In summary, the feedback from the questionnaire can be categorized into two aspects as follows:

- (1) Teachers are not fully prepared for the new curriculum due to the lack of an understanding of the principles, standards, and objectives of the new curriculum.

The success of the execution and the achievement of the new mathematics curriculum heavily depended on behaviours on the executants of the textbooks—teachers. It requires that teachers should prepare the necessary knowledge for the implication of the new curriculum: understand the concept and purpose of the new curriculum. Unfortunately, the result of the investigation in this study shows that their knowledge is far from enough. This can be easily concluded from the answers to the first four items in the questionnaire. Most of the teachers (around 57.4%) can only mention 1 to 3 of the 10 principles of the new curriculum. Even more, 10.8% of the teachers could not mention even one of the principles. Similarly, around 22.1% of the teachers can only mention more than 3 out of 6 purposes of the new curriculum. Unexpectedly, around 30.7% of teachers said that they do not want to know the new concept of teaching advocated by the new curriculum. It attests to the fact that some teachers hold indifferent attitudes toward the curriculum reform. The teaching tradition has restricted their acceptance and enthusiasm toward the new curriculum, and as a result, leads to their shortage of the preparation knowledge for the execution of the new curriculum.

The teaching method used in classroom teaching is actually an indicator of teachers' concept of teaching, whether the concept of teaching changed or not may also reflect the degree to which the new curriculum has influenced. Items 5 and 6 in the questionnaire intended to measure the influence level. Inspection into the answers to these two items reveals that the

teacher-centred “indoctrinating teaching” and student-centred “inquiry teaching” are used almost equally. This seems to show that the new teaching method—student-centred “inquiry teaching” advocated by the new curriculum has started to influence these mathematics teachers. Nevertheless, the subsequent classroom observation and interview show a different perspective: In mathematics classroom teaching, teachers are found to provide more time for students to involve in classroom activities, whereas in practice, teachers’ behaviours are restricted by many factors. They unconsciously follow their traditional teaching method which they are quite familiar with—analyze samples and practice imitatively. This situation implies the distance between the theoretical trend and the practical conduct which actually reflects a shortage of preparation for PCK.

(2) Teachers need to expand their repertoire of teaching strategies and their knowledge of the new added contents in the syllabus.

Teachers’ concept of the new added contents in the textbooks can reflect the quality of their PCK. Items 7 to 12 are designed to inspect this problem. The results indicate that teachers’ attention and understanding of the new contents can hardly ensure the quality of their instruction. On the one hand, they are not quite familiar with the new contents. Sixty-nine percent of them can only dare to say “they know part of the contents about algorithm and statistics.” The research believes that this is not due to their modesty since their answers to the Items 11 and 12 have confirmed it. On the other hand, teachers have a narrow perspective toward the new contents. Nearly 83.5% of them said that, in addition to the textbooks, they can only learn about the new contents through teachers’ reference books. This phenomenon is not due to the shortage of relevant references. For example, “Chinese remainder theorem” is a famous archaic algorithm and there are plenty of references for it, however, 53.4% of the teachers only know this is a kind of algorithm and have no idea about its details; 23.3% of them seem to know nothing about it. Based on all these, it is not strange to find that most of the teachers are not satisfied with their own understanding of the new contents (26.1% of them feel incapable and 6.8% of them reject knowing about the new contents).

The second section of the questionnaire further examined the teachers’ understanding through two typical questions (item 11 and 12). The feedback is notable: (1) only 8 teachers provide complete explanation to the Amours Bertrand problem in the textbook, and (2) among the 176 papers, only 3 know “Gaussian distribution” is another name for “normal distribution.” Although

name is not a serious issue, it does show the other side of the insufficiency of the teachers' relevant knowledge.

Results of Classroom Observation

The researcher interviewed 11 teachers among the 28 who were observed in the classroom. The selection of the teachers is according to their answers to the questionnaire and their enthusiasm for this study. The other 27 teachers who were interviewed are all in-service master students who took classes in Nanjing Normal University. The distribution of the 38 teachers is shown in Table 3.

Table 3
Distribution of the Teachers Interviewed

	Level of School				Title of Teacher			Teaching Age			
	1	2	3	4	Senior	Secondary	Junior	0~5	6~15	16~25	26~
Distribution	6	8	11	13	8	16	14	17	12	5	4

Note. Levels of school: 1= normal school, 2 = two star school, 3 = three star school, 4 = four star school.

The researcher took detailed notes on the classroom observation and interviews, together with voice recording and video tape.

Classroom observations inspected teachers' PCK from the following three aspects: (1) whether they make full use of the knowledge from teaching materials and resources, (2) the way they interpret mathematical concepts and procedures, and (3) their teaching methods and strategies. Among them, the first two focus on the teachers' professional knowledge on mathematics. For effective use of teaching materials, teachers should have a good knowledge foundation so as to evaluate the quality of the materials or resources, and they should have a deep understanding and plentiful knowing of the interpretation of different kinds of mathematical concepts and procedures.

To a certain degree, the results of classroom observation supported and interpreted the conclusions from the questionnaire investment, that is, teachers' professional knowledge is not enough for the new added contents. Most teachers did not fully understand the use of teaching materials. This

could be seen from their performance in class: they read from the textbooks, follow up by giving introductions without further changes; they could not get the key idea of the content or problem; and they are far away from doing the job with ease.

For example, when explaining the differential influence of a geometrical model on probability, they faced the following two questions:

(1) Given an isosceles right-angled triangle ABC , take any point M from its hypotenuse AB . Find the probability of $AM < AC$.

(2) $\triangle ABC$ is an isosceles right-angled triangle. C is the vertex of its right angle. Draw ray CM inside $\triangle ABC$. CM and segment AB intersect at point M . Find the probability of $AM < AC$.

These two questions are quite similar literally, however, their solutions are different. The solution to the first question should be AC/AB , while the solution to the second question should be find a point C' on AB to let $AC = AC'$, then $\triangle ACC'/\triangle ACB$ should be the answer. It is found that most teachers provided the following explanation: for the first question, we should know the distribution of M on segment AB is equiprobable, while for the second question, the distribution of CM inside $\triangle ABC$ is equiprobable. This explanation does not contribute to students' understanding of the questions. In fact, the key to the solutions is: whether the distribution of M on AB equals to the distribution of CM inside $\triangle ABC$. The teachers' answers showed that they did not understand the problems deeply.

The investment in teaching methods and procedural knowledge focuses on the following two aspects: (1) whether they use conceptual knowledge and (2) the knowledge of the use of teaching methods. It is found that nearly 47% of the teachers did pay attention on how to set up an environment for students' problem posing, problem investigation, and problem solving in their lesson plans and they did use various teaching formats. This result is consistent with that of the questionnaires. It does show that some changes have been made of teachers' concept toward teaching, though further strength and promotion are still needed. At the same time, teachers' knowledge of teaching methods, including concrete teaching strategies and lesson organization, are obviously insufficient. The evidence is, sometimes, a good lesson plan could not be carried out smoothly, and finally, they went back to the tradition of comparing with examples and practice accordingly.

Results of Interview

The interview is used to test and verify the thoughts and results from the questionnaire and classroom observation. Since no obviously different conclusions had been made, we will not describe the interview questions or details here. We will only introduce the typical steps.

For the 11 teachers who received both classroom observation and interview, the questions were based on their concept of teaching and strategic knowledge. The interview questions were not very challenging. It was found that the teachers did not catch up with the reform of the teaching concepts. For example, the same question – “What kind of teaching method do you think is more effective?” was asked of each teacher. Most of them replied that “heuristic, induct and encourage students to involve in investigation activities,” however, when asked “What kind of teaching method do you often use to achieve your teaching purpose?” Their answers are relatively consistent: they believe that mathematics learning is based on calculation, deduction, and some other abstract thinking process, and through visual practice and concrete activities they could motivate the students and achieve the teaching effectiveness. Obviously, teachers’ perspective is relatively narrow.

Since the researcher was the lecturer of the 27 in-service master teachers, the interview was more like a test. In addition to the last three items in the questionnaire, the interview questions also covered the investment of their knowing, understanding, and studying of the new added contents in the textbooks. The result showed that only 2 of the teachers can roughly give answers to item 11 and 12. Due to the lack of relevant knowledge, most of the teachers could not give clear and complete explanations. However, almost every one could provide an “appropriate” reason for this, that is, they had just started to teach the new contents and they need time to get familiar with them. Indeed, this is a very important reason. Nevertheless, two points should be made clear: (1) the new curriculum and textbooks have been released for more than two years; corresponding wide range trainings are provided; they should have had enough time to get familiar with the contents; and (2) the familiarity could not be achieved in a short time, because it is a process of accumulation. Thus, more attention should be given to teachers’ PCK under the current curriculum reform; at least, it could be used as one of the important criteria for evaluating the effectiveness of reform.

Conclusions

The results of this study suggested that high school teachers’ PCK is not sufficient to meet the need of current curriculum reform. We found that: (1)

the teachers are not adequately prepared for the new curriculum because of their lack of understanding of the principles, standards, and objectives of the new curriculum; (2) these teachers need to expand their repertoire of teaching strategies and their knowledge of the new added contents in the syllabus; and (3) there is a gap between the teachers' PCK and their classroom practice. That is to say, the quality and quantity of the teachers' "one bucket of water" needs to be improved. For the quantity, they need to learn more about the new curriculum, its perceptions and also get familiar with the new added contents in textbooks; for the quality aspect, they should better understand the essence of the new added contents and improve their knowledge of teaching methods.

It has been sensed at the beginning of the curriculum reform that teachers may lack sufficient PCK. This is the very reason that programs for teacher training are carried out. The question is, even with the training programs, the drawbacks proposed by the curriculum experts still exist. How to reflect on existing problems and design methods to deal with them are some of the crucial problems we faced.

We also discovered through the investigation, most of the teachers held a negative attitude about the modes of current training, especially toward the short-term concentrated training held by normal institutes or government educational organizations. Not to mention that the outside-school training is some typically benefit-driven, it is difficult to promote the development of teachers' PCK only through "theories' spread," "large-class organization" and "one-way monologue."

Of course, it is not fair to attribute the poor situation of teachers' PCK to the low effectiveness of teacher training. The purpose of this study is not to find out whose responsibility it is; instead, we intend to contribute to current training system. We suggest that relevant departments should envisage the current quality of teachers' PCK, motivate teachers' self-development, and develop a new 3-in-1 system with self-training, school-based training, and out-of-school training for teachers' PCK to enhance the foundation for the curriculum reform.

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Appendix

Questionnaire on Mathematics Teachers' Pedagogical Content Knowledge

Title: _____ School: _____ Date: _____

The purpose of this study is to find out your understanding of the new mathematics curriculum standard. The information gathered is used for educational research only and has no relation to the evaluation of your teaching ability and teaching performance in school. All results will be kept strictly secret. Please read each statement carefully and write your answers in the space provided. Thank you very much for your cooperation.

Note: This questionnaire includes two parts. Part I are Multiple-Choice Questions. If they are no suitable choices for you, please specify your responses on the side. Part II are Short-answer Questions. Please indicate your answers in details.

PART 1

- Among the 10 basic principles proposed by the *Mathematics Curriculum Standards for Senior Middle School (Reformed version)*, how many can you specify? ()
 A. All of them B. Around 7 to 9 C. Around 4 to 6
 D. Around 1 to 3 E. None of them
- Among the 6 curriculum objectives in the *Mathematics Curriculum Standard for Senior Middle School (Reformed version)*, how many can you specify? ()
 A. All of them B. Around 4 to 6 C. Around 1 to 3

- D. None of them E. Don't know what the targets are
3. To what extent do you understand about the ideas proposed by the new mathematics curriculum? ()
- A. Very much B. Roughly understand C. Not so much
D. Not at all E. Don't want to understand
4. To what extent do you know about the teaching requirements of the new textbooks? ()
- A. Very much B. Quite clear C. Roughly D. Almost no E. Not at all
5. Generally speaking, you agree that the teacher in mathematics classroom teaching should ()
- A. Explain the knowledge and conduct his students as much as he can so as to avoid detours and mistakes, and lowering learning efficiency
B. Give students more chances to do math by themselves despite they might make mistakes
C. Keep mutual respect and explore hard problems with his students together, and especially encourage them to speak out their own ideas even though some may be wrong
D. Assign the obligatory homework as less as possible to ensure more freedom for students
E. Think that the most effective teaching method is to teach only the essential and to ensure plenty of practice
6. Which of the following teaching methods do you prefer to use in classroom teaching? ()
- A. Inquiry teaching mainly B. Indoctrinating teaching mainly
C. The combination of indoctrinating and exploring
D. Teach only the essential and ensure plenty of practice
E. Sharing teaching and practice equally in a classroom
7. To what extent do you know about the new added contents, e.g., algorithm and statistics, in the new mathematics curriculum? ()
- A. Very well B. Quite familiar C. Roughly know
D. Almost nothing E. Not at all
8. In addition to textbooks, what are the other materials from which you can learn about the new added contents like algorithm and statistics? ()
- A. Teaching reference books
B. Teaching reference books, mathematics curriculum and its interpretation
C. Teaching reference books and mathematics books such as "Number theory", "the History of Mathematics" and so on

- D. Teaching reference books and teaching and researching activities
 - E. Teaching reference books and teaching training
9. How you understand about the “Chinese remainder theorem”? ()
- A. Know its rationale very well
 - B. Quite familiar
 - C. Only know it is an algorithm in the ancient time
 - D. Almost know nothing
 - E. Not at all
10. What do you think of your current knowledge of the new added contents, e.g., algorithm? Is that enough for your teaching? ()
- A. Enough
 - B. Just okay
 - C. Not enough
 - D. Adopt rigid repetition with no expanding or spreading for the students’ capabilities
 - E. Reject them psychically

PART 2

11. What do you know about the question “When you randomly draw a chord in a circle whose radius equals 1, what’s the probability of the event that the length of the chord is greater than $\sqrt{3}$ ”?
12. What do you know about Gaussian Distribution? Beside Gaussian Distribution, what are the other distributions you know about?
13. What are the crucial concepts and basic ideas should be concerned in Senior Middle School mathematics teaching? What is the focal point of statistics teaching?

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