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Elaborating on Relationship between the Instructional Setup and the Students' Opportunity to Learn from the Perspective of Mathematics Tasks Framework in Chinese Classroom

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In the view of Mathematics Tasks Framework (MTF), our research focused on delving into the elaborate relationship between the instructional setup and students' opportunities to learn in Chinese classrooms. We investigated the linkage and dynamic relationship between these two factors according to measures of the instruction quality assessment (IQA) rubrics of forty-six video tapes of Chinese prospective teachers. In addition, we presented a further case study of Teacher Zhang to reveal the process of class and analyze the elaboration of tasks to unfold the routes of effective tasks implementation. Finally, our research outlined suggestions about emphasis on choosing subtle tasks during preparing a lesson, highlighting coherence between tasks while implementing intended curriculum as a result of shrinking the gap between intended tasks and enacted tasks.

Keywords: instructional setup, opportunity to learn, mathematics tasks framework (MTF), MTF Chinese classroom

A lot of evidence indicated that teaching strategies are highly related to achievement of students, which was most likely to be associated with academically rigorous practices in class (Hill, Schilling, & Ball, 2004). Previously, Franke, Fennema and Carpenter (1997) took mathematics tasks framework (MTF) as a popular tool to explore the relationship between the teachers' thoughts and actions of students as they attempt to build understanding of (a) their students' thinking; (b) the mathematics; (c) the teaching.

Ball (1996) presented that we should not only know what teachers know, but also know how teachers teach, and what are their motivations for teaching. Tharp and Ballymore (1988) explicitly distinguished "knowing what it is" from "knowing how to do it" as distinct knowledge types. As a result, more researchers shed light on the realm of the community of teaching and learning, analyzing the trivial scene of class, aiming at uncovering the interior mechanism of mathematics teaching. Some researchers have specified what had happened between teachers and students in classrooms (Franke, Kazemi, & Battey, 2007). For example, research suggested that instruction should include frequent opportunities for students to solve challenging mathematical tasks, to articulate their mathematical reasoning, and to make connections between mathematical ideas and representations (Franke et al., 2007; Hiebert, Carpenter, Fennema, Fuson, & Wearne, 1997). Hill, Rowan and Ball (2005) indicated that there is no direct tool to measure teachers' knowledge. Furthermore, a number of instruments combined descriptions of the nature of classroom work into estimates of teachers' skill and knowledge in teaching such as The Reformed Teaching Observation Protocol (RTOP) (Sawada & Pilburn, 2000), Inside the Classroom Observation and Analytic Protocol (Horizon Research, 2000), and the Learning Mathematics for Teaching: Quality of Mathematics in Instruction (LMT-QMI) (Ball, et al., 2007). However, measuring how teachers instruct in class is estimated as a complicated routine. Arends and Winitzky (1996), and Moll (1992) highlighted the relationship between the quality of teacher-student engagement and students' mastery orientation and avoidance goals. In addition, teachers' and students' practice are supposed to be in a mutually supportive relationship in the classroom (Bishop, Brew, leader & Pearn, 1996). Sharif and Matthews (2017) indicated that multiple pedagogical theories offered the framework to understand the pedagogical features as well as the ineffectiveness or effectiveness of pedagogy, which benefit the specific studies in the divergent education system.

Stein and Smith (1998) proposed the mathematics tasks framework (MTF) in comparison with the tasks as mathematics curriculum, tasks as preparation for teachers and tasks as teaching implementation (Hsu, 2014). Various researchers also indicated that multiple tasks had profound impacts on students' learning opportunities (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996).

Simon (2004) offered further elaboration of reflective abstraction of mathematical concepts using a framework called Reflection on Activity-Effect Relationship (RAER). The framework offered a way to understand how conceptual transitions derived from the learners' activity and reflection. Simon and Tzur (2004) also identified implications of the framework for the design of instructional task sequences. Doyle (1986) defined Cognitive Demand (CD) as what students need to do (e.g., the nature of reasoning) in order to solve a particular problem or, at a broader level, participate in a given activity. Stein and Lane (1996) further argued that one determining factor of the CD of classroom activity, and thus the nature of students' learning opportunities, is the nature of the task that a teacher chooses to use in instruction, or the task as it appears in instructional or curricular materials. In addition, Stein, Grover, and Henningsen (1996) systematically identified the characteristics of mathematics tasks as low-CD and high-CD. Specifically, tasks with low-CD required students to memorize or reproduce facts, or to perform relatively routine procedures without making connections to the underlying mathematical ideas. Tasks with high-CD tended to be open-ended (i.e., a solution strategy is not immediately apparent), which required students to make connections to the underlying mathematical ideas and engage students in disciplinary activities of explanation, justification, and generalization. Based on analyses of middle-grades mathematics instruction aimed at ambitious learning goals， Stein and Lane (1996) found that the use of tasks with high-CD was related to greater student gains on an assessment, leading to high levels of mathematical thinking and reasoning.

Studies (e.g., Hiebert & Wearne, 1993; Stein & Lane, 1996) suggested that increasing focus on cognitively challenging tasks and extended engagement with high-level CDs would be likely to increase students' learning of mathematics. Stein and Smith (1998) listed a comparison of factors associated with the decline and maintenance of high-level CDs. Silver and Smith (1996) suggested that it is necessary to start with a good task in order to provide students with opportunities for engaging in high level thinking, using different approaches, making conjectures, and generalizing. Students often perceive these types of tasks as ambiguous and/or risky because it is not apparent what they should do, how they should do it, and how their work would be evaluated (Doyle, 1986; Romagnano, 1994). Some researchers (Wilson & Goldenberg, 1998; Wilson & Lloyd, 2000; Wood, Cobb, & Yackel, 1991) ensure that students feel successful as they work on more challenging mathematical tasks (Smith, 2000), acquire knowledge from asking questions and providing information (Romagnano,1994), and are provided an appropriate amount of support and structure (Lloyd,1999). Simon (1995) indicated that students' mathematics proficiency might be leveraged through their experiences in solving many cognitively demanding tasks, especially given evidence that Chinese students may learn mathematics effectively by repeatedly working on mathematical tasks.

Ball (1999) indicated that the greater the attention to establish a taken-as-shared understanding of mathematical relationships in the setup, the stronger connection with the quality of the concluding whole-class discussion available. Ball and Bass (2000) inferred that efficient practice aimed to create a practice of mathematics rooted in intellectually honest ways. Brophy (1999) described effective teaching as infused with coherent tasks, structured and connected discussions of the key ideas of mathematics. Hsu (2014) examined geometric calculation with number tasks used in a unit of geometry instruction in a Taiwanese classroom. They identified the source of each task used in classroom instruction and analyzed the cognitive complexity of each task with respect to two distinct features: diagram complexity and problem-solving complexity.

Professional Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 1989) provided a broad foundation by identifying teachers' responsibilities in four areas: (1) setting goals and selecting or creating mathematical tasks to help students achieve these goals; (2) stimulating and managing classroom discourse so that both the students and the teachers can understand explicitly what is being learned; (3) creating a classroom environment to support teaching and learning mathematics; (4) analyzing student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions.

Ball, Sleep, Boerst and Bass (2009) proposed ambitious mathematics teaching as high-leverage practices that teachers can develop. Such practices have the potential to increase student participation and learning as they engage in mathematical activity aimed at rigorous learning goals. Lampert, Beasley, Ghousseini, Kazemi and Franke (2010) analyzed the knowledge and skills involved in supporting each student to develop an increasingly sophisticated understanding of central mathematical ideas; it necessarily requires that teachers teach in response to what students do as they engage in solving mathematical tasks (Kazemi, Franke, & Lampert, 2009; Lampert & Graziani, 2009), characterize this common lesson (Jackson, Shahan, Gibbons & Cobb, 2012) structure in reform-oriented middle-grades mathematics curricula. They thought that the standard mathematics class should include the three-phase lesson: a complex task being introduced, students working on solving the task, and the teacher orchestrating on whole class discussion.

In summary, studies of mathematics learning have not tended to focus on and be grounded upon the theoretical basis of the teaching interventions in order to promote learning, (e.g., Confrey, 1995; Hill et al., 2018; Lampert, 2010; Silver & Stein, 1996). As one of most influential works in the field, the Cognitively Guided Instruction (CGI) Project was based on the belief that instruction should facilitate children's construction of knowledge (Carpenter et al., 1989). Stein (1996, 2000) indicated that, if teachers had less knowledge of interaction, it would be difficult to choose an appropriate task. In order to make practice become the core of the curriculum of teacher education, a shift was inevitably needed from a focus on what teachers know and believe to a greater focus on what teachers do (Ball, 2003).

In this study, our research focused on a more integrated relationship between teaching and learning, focusing on analyzing the linkage between instruction setup and students' opportunities to learn in the perspective of MTF. We identified the mathematics tasks framework (MTF) of prospective teachers, unfolding the visible and dominant empirical connection between instructional setup and students' opportunities to learn.

The questions of the research were listed as:

- What is the linkage among the elements of MTF and what teachers applied in their classrooms?
- How do teachers keep setup maintenance in class and what relevant strategies do they use?

Conceptual Framework

About the Mathematical Tasks Framework (MTF)

Stein (1998) proposed the concept of mathematical tasks framework (MTF). Hsu (2014) explained that the MTF was the sequence of mathematical tasks that provide students with outstanding functional tasks from tasks as curriculum, tasks as setup to tasks as enacted by the teacher and students in the classroom (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996).

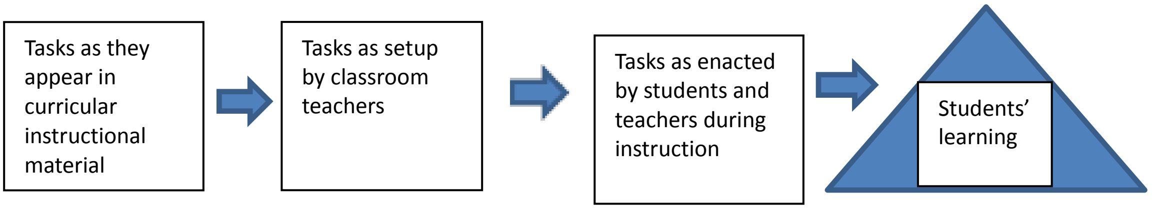

Figure 1. *Mathematics task framework (Modified from Stein & Smith, 1998).*

Piaget (1985) pointed out the concept of disequilibrium under which students' minds were inspired by instruction. Martin and Simon (1997) indicated that while students experienced conflict with individual schemes, a cognitive disequilibrium result would trigger a learning process. Simon, Tzur, Heinz and Kinzel (2004) indicated that disequilibrium may trigger a new conception. Equilibrium, on the other hand, is a state in which one perceives success in removing such an obstacle. In Piaget's terms, it occurs when one modifies his or her viewpoint (accommodation) and is able, as a result, to integrate new ideas into solving the problems (assimilation). Disequilibrium or perturbation is a state when one encounters an obstacle or a result fails to be assimilated. It leads the mental system that seeks equilibrium, that is, to reach a balance between the structure of mind and environment. Its cognitive effect in suitable emotional conditions is that the subject feels compelled "to go beyond his current state and strike out in new directions" (Piaget, 1985, p.10).

Different tasks had the relevant complexity of Cognitive Demand Task (CDT) according to the functional role in class. The divergent complexity of CDT served as motivation for the dynamic transformation from a situation of disequilibrium to equilibrium. Some researchers (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996) reported that in the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) Project, they discovered the importance of matching tasks with goals for student learning.

Stein, Grover, and Henningsen (1996) systematically identified characteristics of mathematics tasks with low and high CD (H-CDT & L-CDT). L-CDT requires students to memorize or reproduce facts or to perform relatively routine procedures without making connections to underlying mathematical ideas. H-CDT tasks tend to be open-ended (i.e., a solution strategy is not immediately apparent), which require students to make connections to the underlying mathematical ideas; and allow students to engage in disciplinary activities of explanation, justification and generalization. The categories of H-CDT and L-CDT can be indicators of the dynamics of instruction in the relevant stage in class. Stein (1996) found that in classrooms where tasks with the potential for H-CDT were assigned, teachers and students often decreased the CD over the course of the lesson. For example, H-CDT could be viewed by a teacher as supporting the development of procedural understanding of a particular skill because of her instructional goals or her knowledge of mathematics. Jackson (2013) suggested that providing students with access to the key ideas of complex tasks while maintaining the CDT is a delicate work. Confrey and Smith (1994) suggested that the increasing exposure to H-CDT and extended engagement in H-CDT could increase students' learning opportunities of mathematics. Teachers' actions and interactions with students and tasks were found to have a major influence on whether CDT was maintained or declined. A series of investigations using data from QUASAR classrooms (Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996) established that the instructional tasks used by teachers in mathematics classrooms were associated with different types and levels of CDT. The complexity of CDT can be specified at several key points across an episode of lesson enactment, and it may fluctuate as the tasks pass through phases during the enactment of a lesson.

In summary, the complexity of CDT in the research referred to the extent to which contextual features occurred and explicit mathematical relationships were exhibited. It leads to disciplinary academic rigor of discussion. Our assumption is that the complexity of CDT would become one of determining factors by which the CDT is maintained over the course of the instruction enactment.

The Definition of Measurement and Development of Rubrics

The researchers in this study focused on the tasks setup and tasks implementation stages and endeavored to identify the instruction quality assessment (IQA) rubrics. Task Potential (TP) reflects the characters of tasks. Tasks setup can be included in contextual features (CF), mathematical relationship (MR), and setup maintenance (SM). Tasks implementation encompassed academic rigor of the discussion (ARD), student linking (SL) and student providing (SP). In general, TP served as the nature of tasks setup, the rubrics of ARD, and SL and SP described students' engagement and activities. The rubrics of MR and SM were used to judge the effect of class. The Expanded IQA focuses on what teachers and students do in the classroom, however, it does not directly measure what students actually learned via instruction (Brian, 2008). Therefore, we explicitly refer to students' learning opportunities provided by teachers, with the assumption that the higher the scores reached on the rubrics, the more likely that students gained chances to learn significant mathematics.

This study selected the four factors of TP, ARD, SL and SP to describe the students' activities in class, which was called Learning Quality Assessment (LQA). It is more likely to consider the following reasons: TP contributed to the extent of difficulties and processes of activities in class. SL and SP could well depict the students' interaction in class. Meanwhile, involving in the procedure of teaching and the setup of CF is significant. MR had to be well exhibited in teaching. Most of all, teachers had to take measures to maintain the CD of the tasks, which was called setup maintenance (As shown in Table 1 and Figure 2).

Table 1

Expanded IQA Rubric Description and Focal Aspects of Instruction

	Rubrics	Focal Aspects of Instruction
	Task potential	CD of the task as it appears in the curricular materials
Student LQA	Academic Rigor of the Discussion	Concerning the disposition of inquiry, the rigor of the mathematics knowledge, and the extent of response to the students.
	Student Linking	Student communication in contribution within the whole-class discussion
	Student providing	Student providing conceptual explanations within the whole-class discussion
Setup	Contextual Features	Building a taken-as-shared understanding of the contextual features of the problem-solving scenario in the task statement.
	Mathematical Relationship	Building a taken-as-shared understanding of the mathematical relationship and ideas in the task statement
	Setup Maintenance	Maintenance of the CD of the task specific to the setup phase of instruction

We added the three-phase structure in the view of students' activities to depict the students' response with the codes of teachers' and students' activities while teachers began to introduce the tasks; we found the students' relevant response to the teachers' conduct. According to our revised mathematical tasks framework, we could analyze the actual relationship between teaching and the process of students' learning.

During the coding, two teachers were responsible for the coding of one factor (see Stein et al., 2000). Then two individuals coded it independently as result to take the mean of two scores as the final scores. Because this was the first time the task-as-set-up rubrics were used on a large scale, we gave reliability information for each rubric separately in Table 2. We attended to a video to test the reliability information for the rubrics according to the standard IQA that we used in our analyses. In general, the reliability scores for the task-as-set-up rubrics did not differ significantly from the reliability scores for the standard IQA rubrics.

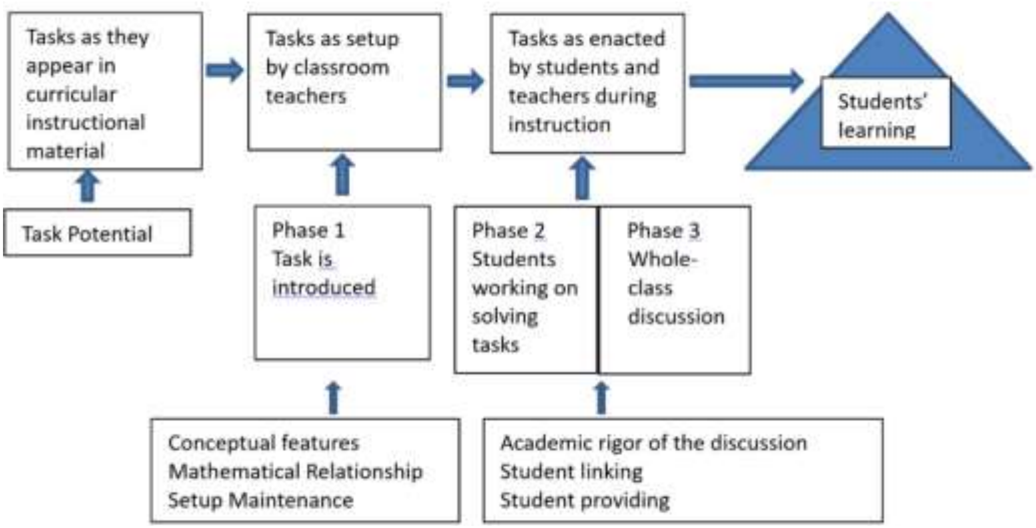

Figure 2. *Mathematical tasks framework (Stein et al., 1997) with three-phase lesson structure.*

Concerning the Role of Instructional Setup

Prospective teachers shed light on the activities of asking questions, and communicating with students smoothly. Their teaching task strategies led to student discussions. Inappropriate instructional setup could lead to decreased expectations in the implementation that followed. Instructional setup played an important role in determining the structure, function and disposition of the tasks. With full preparation and occasional improvisation in class, prospective teachers learned to assign the requirements and the processes of tasks to the students, representing the intentions and goals of the tasks as well as the teacher's elaborated design.

Two Dimensions of Our Research

The research unfolded in two dimensions of research routine. Firstly, the research investigated the quantity linkage among 46 videotapes of mathematics class in MTF. Secondly, our research attempted to unfold how teachers kept setup maintenance in class and what relevant strategies they used by analyzing teacher Zhang's class.

Method

The Design of the Empirical Research

The research process included several aspects. Firstly, the researchers divided every video tape into several independent tasks including warm-up, presentation of a definition, an application section, and ending with a recap. Secondly, scores were given to each task with rubrics of TP, MR, CF, SM, ARD, SL, SP; Thirdly, of the data from the samples were compiled and analyzed.

The Samples of Participating Teachers

Three to four schools in each district were selected to participate in the project. Schools were purposely sampled to reflect variation in student performance and in capacity for improvement within each district. The instruction described in our research was not meant to be good or not good, but was intended to represent their daily teaching state. Our samples consisted of 46 teachers located in the selected four schools: Twelve teachers from School A, eleven teachers from School B, twelve teachers from School C, and eleven teachers from School D. Teachers in our sample averaged 9.5 years of teaching experience. Thirty-one of them were directors of a class in a key elementary school in East China, rewarding a bachelor degree in mathematics education. Twenty-one of them have won many awards, such as "promising beginning teachers at a provincial level" and "excellent trainers of new curriculum", having published several papers and contributed some chapters to reform-oriented mathematics texts and teaching supplemental materials. Twenty-one of them have participated in some projects about mathematics education. Specifically, they exhibited several attributes that appear to be part of the cultural script (Stigler & Hiebert, 1999) of mathematics instruction in Shanghai.

Data Source

A total of 46 videotapes from different sources were viewed. Some of the videotapes were from teaching competitions, while the others were from everyday teaching. Using the interview strategy protocol, we interviewed 46 teachers in videotapes. All transcriptions were recorded as data.

Data collection included video recordings of classroom instruction, interviews with all participants, assessments of teachers' and coaches' mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004), video or audio recordings of professional development sessions, and students' achievement data.

Data Measures Strategy

The research team included three professional researchers trained in scoring the videotapes in order to mitigate the ambiguity of final scores. The final scores were derived from the mean scores from these three researchers.

The inevitable weakness in the data collection in the subjectivity of the scores, despite choosing the mean of three researchers' scores as the final one. In addition, the scores criteria also need further adaptation. These flaws need further research to revise.

Results

In order to unfold the quantitative relevance among elements of MTF in class, we analyzed the codes of MTF from 46 samples and reached some results as follows.

The Significant Impact of TP in Class

As shown in Tables 3 and 4, most teachers chose the medium degree of tasks and got a score 2 in TP. The scores of ARD also reached to relevant medium (a score of 2) in most classes, especially most SL had a score of 1. In addition, the mean score of TP was only 2.7; alternative score of ARD, SL, and SP is low to 2.1, 1.7 and 2.1. It suggested that the scores of ARD, SL, and SP were related to the degree of TP and implied the significance of teachers' choice for tasks.

Table 3

The Mean Scores of Each Rubric for 46 Videotapes

Rubrics	Task Potential	Contextual Features	Mathematical Relationship	Setup Maintenance	ARD	Student Linking	Student Providing
Scores	2.7	1.2	3.2	+(65.2%) -(35.8%)	2.1	1.7	2.1

Table 4

The Comparison of Scores of Rubric TP, ARD, SL, SP

Rubric	Mean	0	1	2	3	4
Task potential	2.7	0	1	18	19	8
Academic rigor of discussion	2.1	9	10	19	6	2
Student linking	1.7	9	30	5	2	0
Student providing	2.1	10	15	15	5	11

As shown in Table 5, when the scores of TP reached 4, or 1, the percentage of maintenance was alternatively high (57.1% and 75.0%). This suggested that the degree of TP has an impact on the setup maintenance. If the degrees of TP are high or very low, it tends to considerably effect teachers' setup maintenance. In contrast, a medium degree of TP poses a challenge to teachers in keeping setup maintenance. For example, setup maintenance was in less than half (29.4%) of the lessons when the scores of TP reached 2. In other words, teachers should place emphasis on a medium degree of TP while teaching, because the setup maintenance for it was more complicated than others.

Table 5
The Comparison of Scores of Rubric TP and SM

Task potential	Setup Maintenance Maintain	Setup Maintenance Decrease	total	Setup Maintenance (%)
4	4	3	7	57.1
3	6	12	18	33.3
2	5	12	17	29.4
1	3	1	4	75.0

In addition, the results show that when the score of TP reached the highest score 4, the scores of SL was only 0 (see Table 4). This indicates that the higher-CD tasks were not certain to have a positive influence on students' learning and understanding. In other words, teachers would be prudent to choose appropriately difficult task to adapt to the students in their classes.

As shown in Table 4, when the score of TP was 2, the amount score of ARD soared to 19, which was the evidence of the assumption that medium difficulty tasks facilitated the discussion of students. As high-level CDT tended to be less structured and more difficult, students were vulnerable to frustration in discussion. Therefore, the medium intricacy of tasks could benefit the engagement of students in class.

Finally, as shown above, scores of SL were significantly related to an increase in scores of TP. The degree of scores of SL could reach the higher scores of 30 while the score of TP is 1. This accounts for the degree of complexity of CDT influencing students' engagement. If the students cannot be actively engaging in class, the teachers need to reflect on the choices of the appropriate tasks in their preparation. The same things could happen in the situation of SP.

In conclusion, the close connection between the level of TP and the quality of students' ARD, SL, and SP revealed the fact that instructional setup could have dynamic influence on students' opportunity to learn. Teachers could not succeed in setup until the appropriate degrees of TP were selected by them. While teachers chose the medium tasks, they should take measures to keep setup maintenance to improve students' ARD, SL and SP.

The Positive Relationship between MR, SL and SP

As shown in Table 7, among the tasks of which CDT was decreased, MR had an ambiguous connection to CF. While about 34.5% got to score 0, only 6.9% got to score 3. It seemed that most teachers paid less attention to the CF in class. Teachers tended to choose tasks with less CF when attending to MR. With the evidence of a close relationship between TP and SL, and MR in prior literature, the higher scores on MR seemed to drive the positive relationship between MR and SL. Similarly, the higher the scores on MR, the more likely to receive higher scores on ARD for students. However, there is a divergent situation for the aspect of scores of CF.

Table 6

Task Potential and Setup Maintenance (n=46)

Task potential	maintain	Decrease	total	% Maintain
4	4	4	8	50.2
3	6	13	19	33.3
2	6	13	18	30.6
1	1	0	1	100
Total	17	29	46	37.0

Table 7

CF and MR Scores in Setup Maintenance

Score	0	1	2	3	4
CF	34.5%	24.1%	20.7%	6.9%	13.8%
MR	10.3%	20.7%	37.9%	20.7%	10.3%

A conception-based perspective is grounded in a view of mathematics as a connected, logical and universally accessible part of an ontological reality (Wood, 1998). It is embedded in the nature of the mathematics problems presented in class and the way in which the problems were worked on the students (Hiebert, Douglas, 2007). In view of conception-based perspective, students were interested in more realistic and complicated mathematics problems, and shared their strategies and solutions orchestrated by teachers. The greater the attention to establish a taken-as-shared understanding of mathematical relationships in the setup, the stronger the possibility of the quality of the concluding whole-class discussion would have. During the discussion, there was little filtering by teachers that mathematics helped to illustrate, or any attempt to highlight those ideas. The teaching strategy influenced the engagement of students; thereby, contributing to the ensuing robust discussion by students.

In summary, the linkage relationship between MR and CF, SL, SP suggested that MR played a significant role in class. It suggested necessity for teachers to provide students with explicit mathematics knowledge and ideas.

The Complicated Relationship between SETUP Maintenance and ARD, SL, SP

The intricate setup contributed to building the productive routines of teaching. According to Schoenfeld (2011), there is a widespread assumption that discourse, norms and relationship are three elements of productive teaching. Setup is associated with the creation of structure in class; and as a result, the intricacy of teaching is mostly imbedded in the setup of instruction.

Table 8
The Relationship between Setup Maintenance and ARD

ARD	Setup Maintenance		Total	%
	Maintain	Decrease		maintain
4	4	2	6	66.7
3	7	12	19	36.8
2	5	11	16	31.3
1	3	2	5	60.0

As shown in Table 8, the relationship between Maintenance and ARD was ambiguous and complicated. There was no strikingly positive relationship between them. It implied that ARD was not a determining factor, but had some influence on SM. SM served as the predictor of effective teachers' tasks design. As a result, we were unable to judge the effect of SM until ARD, SL, and SP were combined into our thinking.

Further research concerning the relationship between SM and SL also revealed the complicated connection between them. As shown in Tables 9 and 10, it also implied that SP could be taken as an emerging and improvising process. Sometimes planned tasks are not adapted to the process of teaching. Teachers need to adjust their teaching in terms of SP in class.

Table 9
The Relationship between Maintenance and SL

SL	Setup Maintenance		Total	% maintain
	Maintain	Decrease		
4	2	5	7	28.6
3	5	13	18	38.5
2	6	11	17	35.3
1	2	2	4	50.0

As shown in Table 10, when the percentage of setup maintenance reached 75.0%, the score of ARD was only low to 1. It implied that the dynamic relationship between setup maintenance and students' opportunity to learn occurred in the limited condition. In other words, the setup maintenance never became the distinctive element; there existed other multiple factors. In addition, there existed the complicated and dynamic relationship among Setup Maintenance and ARD, SL and SP.

As Ball (2003) indicated, deliberate structure and sets in preliminary class contributed to high level of possibilities of learning for students. The productive setup can contribute to the enactment of intended routines including contribution of implementation of tasks, the motivation of whole-class discussion and the comprehensive of mathematics. The appropriate intended tasks were not taken as full condition but as prerequisite one. The Chinese class was full of uncertainty and improvisation. Setup maintenance was related to ambiguous teaching knowledge.

Take for example, our research generalized the distinct actions between setup maintenance and decline as shown in Table 11.

Table 10

The Relationship between Setup Maintenance and SP

SP	Setup Maintenance		Total	% maintain
	Maintain	Decrease		
4	2	5	7	28.6
3	8	10	18	44.4
2	9	8	17	52.9
1	3	1	4	75.0

Table 11

Comparison Elements to Influence the Maintenance or Decline of Setup

Processes Associated with the Decline of setup	Processes Associated with the Maintenance of setup
Routinizing problematic aspects of the task	Scaffolding students' thinking and reasoning
Shifting the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer	Providing a means by which students can monitor their own progress
Providing insufficient time to wrestle with the demanding aspects of the task or so much time that students drift into off-task behavior	Modeling of high-level performance by teacher or capable students
Engaging in high-level cognitive activities is prevented due to classroom management problems	Pressing for justifications, explanations, and/or meaning through questioning, comments, feedback
Selecting a task that is inappropriate for a given group of students	Selecting tasks that build on students' prior knowledge
Failing to hold students accountable for high-level products or processes	Drawing frequent conceptual Connections

In conclusion, there was no salient connection among SM and ARD, SM and SL, or SM and SP. In other words, SM was associated with integration of ARD, SL and SP. As a result, teachers should consider their class in the view of all aspects of ARD, SL, and SD.

Discussion

This study analyzed the linkage between instruction setup and students' opportunities to learn in the perspective of MTF. It also identified the mathematics tasks framework (MTF) of prospective teachers by unfolding the visible and dominant empirical connection between instructional setup and students' opportunities to learn. The results of the data analysis in this study provide evidence of the following findings:

The appropriate tasks had a great impact on students' opportunities to learn, so teachers should deliberately choose tasks and pay attention to consistency among tasks, in consideration of the complexity of CDT. Only through those chosen with suitable CDT to the students could teachers offer students chances to learn. This finding is supported by a study of Ball (2010) who indicated that the work of teaching is not only unnatural but also intricate, and involving high levels of coordination. The finding also support Ball (2010)'s study concluded that specifying the content of a practice-focused professional curriculum involves careful analysis of the core tasks of teaching. The finding is consistent with findings of past studies by Ball (2010) and Hill (2018), which indicate that more accurate knowledge of what students know and do not know also may assist teachers in other ways, for instance in planning to reteach content that has not been mastered and in designing tasks and instruction that intentionally elicits typical student mistakes with content.

As mentioned above, Chinese prospective teachers applied multiple representations and teaching with variation to maintain consistency among tasks. They struggled to keep connection between the tasks package by using conceptual features in teaching. Effectiveness in consistency could create more space for learning for students. This finding is supported by a study of Ball (2010) who suggested that teaching converges with intuition, as active recognition and incorporation of student mathematical thinking in the classroom should provide teachers with additional information on their students' content mastery and information that likely assists teachers in asking appropriate questions and using their thinking. The finding is also consistent with findings of past studies by Ball (2010) and Hill (2018), which suggested that the quality of class would be related to accuracy, specifically remediation of student misconceptions and use of student productions.

Setup maintenance was closely associated with ARD, SL and SP in class, whether PT was high or low. Multiple strategies of using specific communication language and teaching with variation contributed to the degree of consistency among tasks implementation and helped to keep Setup maintenance. The finding supports Ball (2010)'s study that accuracy also predicted teachers' remediation of students' mathematical errors during class when excluding the control for MKT/MTEL. The finding is also consistent with findings of past studies by Ball (2010) and Hill (2018) emphasized that effect of teacher knowledge of students on student outcomes were independent of teacher knowledge of subject matter.

Conclusion

The aims of this study were to determine the linkage among the elements of MTF, identify what teachers apply in their classrooms and how teachers keep setup maintenance in class. We investigated the linkage and dynamic relationship between these two factors according to measures of the instruction quality assessment (IQA) rubrics of forty-six video tapes of Chinese prospective teachers. We drew important conclusions from this work included choosing subtle tasks during preparing a lesson, highlighting coherence between tasks while implementing intended curriculum as a result of shrinking the gap between intended tasks and enacted tasks. The current findings added to a growing body of literature on teaching practice and students' performance in class, providing more elaborated materials involving Chinese education. Therefore, our recommendation is involving in such aspects as (a) Teachers need to pay more attention to and be deliberately selective of tasks in lesson preparation, (b) Teachers need to pay more attention to the coherence of tasks while implementing intended curriculum, and (c) Teachers should take measures to keep setup maintenance.

Because of the restriction of time and financial factors, we only chose 46 videos for the samples, which are likely to less reflect the authentic situation of our research problems. In addition, our research code could have more space to improve and our research tools could be more acute to analyze the data. Finally, as the sophisticated relationship between teaching and learning, we left some continuous research topics for the next work, for example, what is the significant factor affecting students' mathematics thinking and applying it after class. We believe our research could call for more people to focus on the detail of mathematics class and offer more advantageous strategies.

In conclusion, this study highlighted the significance on choosing subtle tasks during preparing a lesson, highlighting coherence between tasks while implementing intended curriculum as a result of shrinking the gap between intended tasks and enacted tasks.

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