

Mathematical Modeling: Essential for Elementary and Middle School Students

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This article argues for the implementation of mathematical modeling activities within the elementary and middle school years in United States. The article also discusses challenges associated with mathematical modeling from both teachers' and students' perspectives. Examples of authentic modeling tasks are illustrated in this article to indicate the relevance and importance of mathematical modeling and show that elementary and middle school students are capable of engaging in such modeling tasks. It is argued that mathematical modeling tasks are powerful vehicles for developing quantitative reasoning, problem-solving skills, and modeling competencies in the early school years. The article demonstrates that mathematical modeling tasks encourage the development of a wide range of mathematical practices and 21st century learning skills that are useful for real-life situations and today's world.

Key words: Mathematical Modeling; Modeling Process; Authentic Modeling Tasks; Cognitive Demand; Mathematical Practices; Mathematical Processes; Problem-Solving

Researchers, professional organizations, and mathematics education standards have emphasized the need and relevance of mathematical modeling and especially in early school years (Blum, 1995; Blum & Borromeo Ferri, 2009; CCSSI, 2010; Gainsburg, 2008; Lesh & Doerr 2003; Pollak, 2003; NCTM, 2000). Recent studies indicate that young students can make important mathematical and social improvements from working on authentic modeling tasks (Chan, 2008; Doerr & English, 2003; English & Watters, 2004). The National Council of Teachers of Mathematics (NCTM) specified that instructional programs for K–12 should enable students to “use representations to model and interpret physical, social, and mathematical phenomena” (NCTM, 2000, p. 67). Additionally, the Common Core State Standards for Mathematics (CCSSM) emphasizes the use of mathematical modeling: “Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace” (CCSSI, 2010, p. 7).

However, traditional word problems that young students' typically meet in schools lack modeling processes and practices. Studies have shown that modeling practices only play a minor role in everyday teaching of mathematics

in most K–12 schools (Blum & Borromeo Ferri, 2009; Doerr & English, 2003; Pollak, 2003; Zawojewski, 2010). This is not unique in the case of the United States. Anecdotal experiences show that most U.S. elementary and middle school teachers barely use modeling tasks in their classrooms. Although, the CCSSM addresses mathematical modeling as a mathematical practice for all grades, it is limited to only high schools as a conceptual category (CCSSI, 2010). Moreover, in most classroom practices all over the world, mathematical modeling still has a far less prominent role than is desirable (Blum & Borromeo Ferri, 2009; Boaler, 2001)

Consequently, this article argues for the implementation of mathematical modeling as a conceptual category in the CCSSM for elementary and middle schools. As future leaders and members of the world's workforce, young students need to develop the essential components of mathematical modeling. Elementary and middle schools are the educational environment where students should begin a meaningful development of modeling processes and practices (Langrall, Mooney, Nisbet, & Jones, 2002). The article provides a conceptual framework for understanding the modeling process and implementing modeling tasks appropriate for elementary and middle school students. In addition, the article discusses possible challenges with mathematical modeling tasks. It is argued that modeling tasks are powerful vehicles for developing important mathematical practices, problem-solving processes, and 21st century learning skills in the early school years.

What are Mathematical Models?

Throughout the literature on mathematical modeling and models, experts have produced various definitions for the concept "model." English, Fox and Watters (2005) viewed models as conceptual systems used to construct, interpret, explain, and mathematically describe a situation. Alternatively, Lesh and Fennwald (2010) explained that a "model is a system for describing (or explaining or designing) another system for some specific purpose" (p. 7). According to Lesh and Doerr (2003), models are meaningful conceptual systems that foster conceptual understanding and mathematization. Thus, models are descriptive solutions to real-world problems composed of one or more representations. If the behavior predicted by our model does not reflect what we see in the real world, it is the model that needs to be changed and not the world (Dym, 2004).

Experts in mathematics education identify models as tools manipulated in mathematics or statistics to interpret real-world situations (Lesh & Doerr, 2003). Models are among the most important cognitive objectives of mathematics and statistics instruction. Therefore, NCTM (2000, 2014) and CCSSI (2010) emphasize the use of representations to model, interpret, and communicate real-world situations. Although models are significant in the teaching and learning of mathematics, its practice through traditional word

problems present students with skills limited in critical thinking and conceptual understanding (Sriraman & English, 2010). Nevertheless, studies have shown that mathematical modeling presents an alternative approach adequate for solving real-life scenarios that promotes and enhances critical thinking, classroom discourse, and conceptual understanding (Lesh & Zawojewski, 2007; Sriraman & English, 2010; Zawojewski, 2010).

What is Mathematical Modeling?

Researchers in the mathematics education community have offered different definitions of mathematical modeling. Modeling requires translations between reality and mathematics, where students are challenged to study a situation using models and testing that the solution makes sense in the context of the real-world situation (Swetz & Hartzler, 1991). Pollak (2003) explored deeper into this action, stating that mathematical modeling involves (a) a situation in the real-world, (b) making certain assumptions, (c) utilizing a model to obtain a mathematical formula, and (d) applying mathematical procedures to the formula to achieve reasonable real-world answer. Thus, mathematical modeling is a real-life task, which involves mathematical practices and processes such as critical thinking, high cognitive demand, and communication.

Mathematical modeling presents students with realistic problem solving experiences that involves critical thinking skills. Modeling tasks requires strategizing, using prior knowledge, and testing and revising solutions in a real-world context (Greer, 1997; Lesh, Doerr, Carmona, & Hjalmarson, 2003). Thus, the process of formulating and improving a mathematical model to represent and solve real-world problems describes mathematical modeling. On a similar note, mathematical modeling is the process of translating between the real world and mathematics in both directions (Blum & Borromeo Ferri, 2009). The modeling process is building a link between mathematics as a way of making sense of our physical or social world and mathematics as a set of formal structures and representations (English et al., 2005; Greer, 1997).

The high cognitive demand of modeling tasks, pose a challenge in constructing a mathematical model for real-world situations. Therefore, with an explicit modeling process, students are able to understand the process and develop strategies to solve the problems. The modeling process involves observing a situation, conjecturing relationships, applying mathematical analyses, obtaining mathematical results, and reinterpreting the model in the real-world (Lingefjård, 2004). According to the CCSSI (2010), the modeling process involves posing a problem, formulating a model, analyzing and performing operations, interpreting results, validating the conclusion, and reporting the conclusion with explanations.

The reporting and explaining processes involved in the modeling process provide students with the opportunity to communicate and interact verbally with their peers. In addition, the modeling process helps students to

engage in social learning contexts where they can verbalize their thinking and make explicit their knowledge of content and process. Moreover, the modeling process provides students the opportunity to assess each other's work (peer assessment) and formative assessment by teachers is able to occur as well. English et al. (2005) affirmed that modeling tasks engage young students in constructive dialogue and debate with their peers as they share ideas, question one another's claim, justify and refute arguments, and resolve conflicts.

Benefits of Mathematical Modeling for K-8 Schools

Although modeling has been limited to older students, it has many immediate benefits at the early school years. Research has found mathematical modeling to be beneficial in teaching mathematics. For mathematics to be meaningful and relevant to students, experiencing modeling activities at the early school years is a necessity (Blum, Galbraith, Henn, & Niss, 2007; English, 2007). Studies have shown that when students get the opportunity to engage in authentic modeling tasks, they are more engaged in learning and it improves students' achievement (Boaler, 2001; Pollak, 2003). Thus, modeling activities can be fruitful grounds for young students' representational, mathematical, and social development.

The richness of modeling experiences provide young students with the opportunity to engage in tasks that match their current conceptual understanding while simultaneously presenting opportunities for challenge and growth (Flevaris & Schiff, 2013). Mathematical modeling brings an important perspective to K–12 mathematics education in that the real world is not just a context to highlight the value of mathematics; rather, both the real world and mathematics are taken seriously (Pollak, 2007). The author classifies the benefits of mathematical modeling into four main domains: relevance, cognitive demand of task, critical thinking skills, and classroom discourse.

Relevance

Mathematical modeling tasks allow students to appreciate the relevance and usefulness of mathematics to individuals as well as the society (Pollak, 2003). Modeling is increasingly recognized as a powerful vehicle for not only promoting young students' understanding of a wide range of key mathematical and scientific concepts, but also for helping them appreciate the potential of mathematics as a critical tool for analyzing important issues in their lives, communities, and society in general (English, 2007; Greer, Verschaffel, & Mukhopadhyay 2007). Recent research indicated that elementary and middle school students value the importance and utility of mathematics by developing and constructing their own mathematical models through sense making of real-world problem scenarios (Boaler, 2001; Chan, 2008).

Through mathematical modeling, students recognize the beauty of mathematics and see its value and usefulness (Sriraman & English, 2010).

Hence, it motivates and supports the interest of young children in mathematics. The use of mathematical modeling in classrooms mostly eliminates questions from students regarding “what good is this topic or stuff?” (Pollak, 2003). Research has shown that modeling promotes students understanding of a wide range of important mathematical concepts. Modeling when introduced early and often can become a unifying force and help keep students together through the usefulness of mathematics (Pollak, 2007).

Modeling activities create opportunities for young learners to perceive mathematics as useful and applied, rather than abstract and isolated (Blum et al., 2007; Greer et al., 2007). It is increasingly recognized that modeling provides students with a “sense of agency” in appreciating the potential of mathematics as a critical tool for analyzing important issues in their lives, their communities, and the society as a whole (Greer et al., 2007). Modeling engages and supports students’ interest in mathematics plus makes mathematics more meaningful and relevant (English & Watters, 2004; Pollak 2003; Pollak, 2007). Additionally, modeling provides powerful and effective ways to help young students become better problem solvers and able to use mathematics in real-life situations beyond school (Lesh & Zawojewski, 2007).

Cognitive Demand of Task

Unlike traditional word problems, modeling tasks require high cognitive demand. The high cognitive demand takes students beyond basic problem-solving where they derive meaning from symbolically described word problems, to authentic situations that need interpretation and description in mathematical ways (Lesh & Hager, 2001). Elementary or middle school students not only have to work out how to reach the goal state but also have to interpret the goal itself as well as all of the given information. Modeling provides young students the opportunities to elicit their understanding and to make sense of the situation in ways that are meaningful to them. Thus, through modeling children’s conceptual understanding and competencies in mathematics are better developed due to the high cognitive demand of the task.

Modeling tasks are of high cognitive demand and they lead to mathematical explorations and generalizations. All tasks are not created equal—different tasks require different levels and kinds of student thinking (NCTM, 2014; Stein, Smith, Henningsen, & Silver, 2009). Students learning gains are greatest in classrooms where instructional tasks are of high cognitive demand and students get the opportunity to explain their thinking and reasoning (Stein et al., 2009). Additionally, the high cognitive demand of modeling tasks offers students richer learning experiences and multiple points of entry in solving real-life problems. A high cognitive demand task helps to improve mathematics teaching and eventually leads to greater gains in students’ achievement (Stigler & Hiebert, 2004).

Critical Thinking Skills

Modeling tasks help students to develop critical thinking skills, think outside the box, ask the right questions, and make informed choices (Flevaris & Schiff, 2013). Mathematical modeling affords students the opportunity to develop the capability to use models to interpret and explain complex systems, to develop representational fluency, to reason mathematically in diverse ways, and to use sophisticated technological tools and resources (Lesh & Heger, 2001). This helps the students to think critically through the problem before tackling it. Because modeling problems do not have a specific structure to its solution, it helps the students to become critical thinkers in the solution process. If implemented properly, modeling tasks build and improve students' procedural understanding by engaging them in thought-provoking, multifaceted complex problem (Lesh & Doerr, 2003; Zbiek & Conner, 2006).

Mathematical modeling fosters among students metacognitive skills, conceptual understanding, competencies, creative and innovative abilities, and socio-cultural role of mathematics (Blum, 1995). Research shows that modeling promotes students' understanding of a wide range of key mathematical concepts and "should be fostered at every age and grade . . . as a powerful way to accomplish learning with understanding in mathematics" (Romberg, Carpenter, & Kwako, 2005, p. 10). In a longitudinal study involving elementary school students, the research showed significant improvement in the students' development of metacognitive and critical thinking skills. The researchers found that the modeling problems got the students engaged and provided opportunities for the students to express their ideas and thinking in multiple representations (English & Watters, 2004).

Classroom Discourse

Mathematical modeling experiences are ideal to promote communication and teamwork through social experiences. Modeling tasks allow young students' to explore collaboratively that is increasingly important for 21st century learning. In a study conducted by English et al. (2005), students explored cyclones in Australia to design a warning system in a project. The modeling process served as a connection between mathematics as a way of making sense of the physical or social world and models. The students debated, engaged in constructive discussion, and shared ideas when working on creating models and representations of their data.

Mathematical modeling fosters teaching styles that promote students active approach to constructing their own learning through dialogue, discussion, and commenting on each other's work. Students are more empowered and they become active participants when engaged in modeling activities (Redmond, Sheehy, & Brown, 2004). Modeling activities encourage collaborative work and they help students to develop explicitly sharable products that are subject to scrutiny by their peers. The communication processes inherent in modeling

activities play an important role in student's social, as well as mathematical development. Research has shown that when students engage in mathematical modeling the impact on classroom discourse is positive and healthy (Flevaris & Schiff, 2013; Redmond et al., 2004).

Possible Challenges with Mathematical Modeling

Notwithstanding the benefits that mathematical modeling brings to the teaching and learning of mathematics, there are some forces that pose as challenges. Described below are typical challenges that researchers and mathematics educators' face as they explore mathematical modeling: The first challenge is about real-world knowledge and unfamiliar phenomena. In order for mathematical modeling to work effectively in the classroom, teachers have to give up most of their traditional ways of engaging their students. Thus, to model phenomena from outside of mathematics, one has to understand those phenomena. According to Blum and Borromeo Ferri (2009), the main reason why teachers have difficulty with modeling is that real-world knowledge is necessary and teaching becomes more open and less predictable.

The second challenge is about the cognitive demand of modeling tasks and classroom practices. There is a gap between the goals of the educational debate and everyday practice in the classroom because modeling seems difficult for most teachers (Blum & Borromeo Ferri, 2009). This places a burden on some mathematics teachers to understand certain phenomena that they normally are not required, therefore making it demanding. There is also the fear on teachers' part that students cannot solve modeling tasks due to the high cognitive demand. This is because mathematical modeling is inseparably linked with other mathematical competencies such as reading, communicating, designing, and applying problem-solving strategies, which emphasizes high cognitive skills. Maass (2010) explained that most teachers assume that most students find modeling difficult or challenging, hence they seldom implement them in classroom practices.

The third challenge involves the complex nature of modeling tasks and the need for critical thinking skills. Most teachers find working with modeling task complicated and time consuming; hence, students do not get enough time and the opportunity to process their thinking skills. Research shows that it is a challenge to teach mathematical modeling in most traditional classrooms because of students' attitude (Lingefjård, 2004). Moreover, most schools curricula are driven by accountability and standardized testing; hence, most textbooks developed for elementary and middle schools place less emphasis on mathematical modeling tasks or activities. Other studies show that most teachers rarely use modeling tasks in their classrooms because of the time constraint and their perception that modeling tasks are complex (Blum & Borromeo Ferri, 2009; Maass, 2010).

The fourth and final challenge involves assessment, communication, and social skills in mathematical modeling. From anecdotal experiences, most textbooks for elementary and middle schools do not address the inherent communication skills associated with modeling tasks. Additionally, there are no clear and concise rubrics developed to help teachers' assess modeling tasks. Consequently, most teachers find assessing modeling task(s) complicated and time consuming (Lingefjård, 2004). Moreover, because accountability and standardized testing drive most schools curricula, most classroom practices do not allow their students much time to work on modeling problems, where they can discuss, critique, or scrutinize their peers work. This behavior deprives most students from developing their communication and social skills needed for today's world.

Implementing Mathematical Modeling Tasks

Despite the challenges associated with mathematical modeling tasks, this section demonstrates ways of implementing modeling tasks in the classroom. To engage elementary and middle school students in mathematical modeling, authentic modeling tasks that depict real-world scenarios are employed (English & Watters, 2004; Lesh & Doerr, 2003). By engaging in such model tasks or activities, students get the opportunity to learn about other mathematical concepts as well as other interdisciplinary subjects, thus broadening the scope of disciplines towards greater meaningfulness in problem-solving. Two typical classroom-modeling tasks are demonstrated to underscore the relevance, usefulness, and practicality of mathematical modeling.

The first modeling task called the "Kidney Beans" was adapted from the "Butter Beans" problem (English & Walters 2004, p. 338). Students have to examine two tables of data displaying the weight of kidney beans after 7, 9, and 11 weeks of growth under two conditions, sunlight and shade (See Figure 1). Using the data in figure 1, students have to (a) determine which of conditions was better for growing kidney beans to produce the greatest crop; (b) predict the weight of kidney beans produced on week 13 for each type of condition; and (c) in a letter to farmer Jones, the students are to outline their recommendation and explain how they arrived at their decision.

<i>Shade</i>				<i>Sunlight</i>			
<i>Kidney Bean Plants</i>	<i>Week 7</i>	<i>Week 9</i>	<i>Week 11</i>	<i>Kidney Bean Plants</i>	<i>Week 7</i>	<i>Week 9</i>	<i>Week 11</i>
<i>Row 1</i>	<i>5 kg</i>	<i>9 kg</i>	<i>15 kg</i>	<i>Row 1</i>	<i>9 kg</i>	<i>12 kg</i>	<i>13 kg</i>
<i>Row 2</i>	<i>5 kg</i>	<i>8 kg</i>	<i>14 kg</i>	<i>Row 2</i>	<i>8 kg</i>	<i>11 kg</i>	<i>14 kg</i>
<i>Row 3</i>	<i>6 kg</i>	<i>9 kg</i>	<i>12 kg</i>	<i>Row 3</i>	<i>9 kg</i>	<i>14 kg</i>	<i>18 kg</i>
<i>Row 4</i>	<i>6 kg</i>	<i>10 kg</i>	<i>13 kg</i>	<i>Row 4</i>	<i>10 kg</i>	<i>11 kg</i>	<i>17 kg</i>

Figure 1. Data presented for the “Kidney Beans” problem. Students are advised that the farmer had grown 4 rows of kidney beans under two light conditions (adapted from English & Watters, 2004, p. 338).

The second modeling task called the “Rock Concert” was adapted from the Organisation for Economic Co-operation and Development (OECD, 2003) assessment framework of the Programme for International Student Assessment (PISA). The task is framed this way:

For a rock concert, a rectangular field of size 100m by 50m was reserved for the audience. The concert was completely sold out and the field was packed with all the fans standing. Estimate (a) the total number of people along the entire field and (b) the total number of people attending the concert, justify your results.

The first modeling task gives students the opportunity to interpret and understand data presented in various representational formats. Students also get the chance of understanding the mathematical concepts of patterns, change, and rate of change inherent in the mathematical task. The modeling task provides students with rich opportunities in expressing their ideas in multiple representations. The second modeling task gives students the opportunity to utilize or connect different mathematical concepts such as unit conversions, estimation, perimeter, and area in solving an authentic problem.

The two modeling tasks help students to appreciate and value the importance of mathematics. Students are able to realize and recognize how mathematics and the real-world are connected. The real-life application of both tasks presents an opportunity to support and motivate young students’ interest in mathematics. By engaging students in these model-eliciting activities, they are involved in problem solving since they have to move from the givens towards achieving their goals. Thus, modeling tasks help the students to develop conceptual understanding through the mathematical processes. The two model-eliciting activities help students develop important mathematical processes, practices, and generalizable concepts.

Since the two model tasks are authentic and of high cognitive demand, it is consistent with the notion that a problem should indeed pose an obstacle, whereby the students have no immediate solution (Zawojewski, 2010). The

model-eliciting activities involve tasks of high cognitive demand and 21st century learning skills. This requires the students to exercise thinking that is beyond what they normally do in solving traditional word problems. Thus, the modeling tasks help the students to be critical thinkers, more creative, and innovative. The modeling tasks provide the students the opportunity to engage in social learning contexts where they can verbalize their thinking and make explicit their knowledge of content and process.

The two modeling tasks help the students to develop their communication skills, become active participants, and well engaged in the lesson. By working on the modeling tasks, students have to draw on their own specific mathematical knowledge and transfer such knowledge in light of real-world context. Thus, the students through the modeling process have to hypothesize, conjecture, compare, interpret, analyze, explain, and justify in making their thinking known (Lesh et al., 2003). Thus, the students continue to modify, refine, and extend their conceptions to arrive at a reasonable and justifiable conclusion.

Conclusion

The multifaceted nature of modeling tasks makes them ideal channel for advancing young students learning in many directions, particularly in posing and solving problems with real-world scenarios (English et al., 2005). Modeling provides rich learning experiences for elementary and middle school students. Mathematical modeling facilitates young students' collaborative problem-solving efforts as well as fosters their mathematical thinking and learning. Modeling fosters among students general creative and problem solving attitudes, and trains students as to how to apply mathematics to other situations. Blum (1995), argued that mathematical modeling in schools facilitate learning, prepares students to use mathematics in different areas, develop general competencies, and comprehension of the socio-cultural role of mathematics.

Studies have shown that mathematical modeling serves many everyday situations, emphasizes the usefulness of mathematics, and connections to other disciplines (Gainsburg, 2008). Mathematical modeling incorporates the ideals of the 21st century learning and skills. Through modeling, mathematics can be valued by the society and students confidence in the worth and learning of mathematics for a career becomes more meaningful and relevant. The use of mathematical modeling at the earlier grades brings back that aspect of mathematics that greatly reinforces the unity of the total mathematical experience and support students interest in mathematics (Pollak, 2003, 2007).

Mathematical modeling ensures that young children have practical experience applying mathematics skills to real-world scenarios. The intrinsic social experiences inherent in modeling help young students develop powerful collaborative skills that are increasingly important for today's world. The author hopes that the reader has attained an elevated awareness of the current

literature regarding how mathematical modeling positively influences students learning, achievement, and socio-cultural development of young children. Preparing students' for responsible citizenship and participation in societal developments requires them to develop modeling competencies. It is imperative that all researchers, policy makers, and the mathematics education community understand the place and purpose of mathematical modeling.

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